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A New Technique To Handle Local Minimum For Imperfect Potential Field Based Motion Planning

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Abstract

Random walk like techniques have been used to help potential field based motion planning techniques to escape from local minimum configurations. However, the associated cost can be large for some applications which require smoothing to take out the incoherent motion steps that are discovered through the random walks. In this paper, we present a new technique to reduce or eliminate the need for random walks while improving performance. We discuss characteristics in application which can be a potential candidate to benefit from this new technique.

1. INTRODUCTION

Motion planning involves making decisions over a large solution space. The complexity of the problem has been studied extensively (see [6] and [7]). But practical methods proposed to date mainly solve limited, simplified problems. Basically, existing solutions fall in: 1) skeleton 2) cell decomposition, and 3) potential field categories. The first two approaches become very difficult to implement for applications that involve large degrees of freedom (dof) (see [3] and [10]). Potential field based methods, while easier to implement, have been shown to be effective for large dof problems (see [1] and [5]). They make use of an artificial navigation force which is calculated according to the environment to facilitate the exploration of the solution space called configuration space (or c–space). An ideal potential value is defined over each point in the c–space so that a straight forward search approach (such as a steepest decent) will yield a path to reach a goal configuration q_f. In practice for problems with large dof, the field used is usually non–parametric, discretized representation of the free space, superimposed with a distance function indicating how far a given configuration is relative to q_f.

For volumetric objects, an arbitration function is necessary to compose the overall effect of a set of sampled potential values calculated on selected points associated with the object. The complexity of such an overall field is a function of the environment, the moving object, and the arbitration function. Although a point based potential function may be monotonic, for a volumetric object, the composed effect may lead into a *trap* configuration – or a *local minimum*, where all its neighbor configurations have larger potential values.

2. THE PROBLEM

Some attempt has been made to produce local–minimum free potential field for volumetric objects. Given obstacles in symbolic expressions, Rimon showed that in a transformed space a local–minima–free potential field can be computed to facilitate a simple traverse to a unique minimum at q_f [9]. Unfortunately, such perfect obstacle information requirements are rarely met in reality. For all practical purposes, local–minima are present in potential fields for daily applications of motion planners that deal with complex problems when polyhedral models are used instead.

Accordingly, potential field guided, search based motion planning algorithms employ explicit heuristic strategies to escape local minima. For example, a local minimum configuration can be escaped from through random walks [1]. Let configuration q be the local minimum configuration, or $p(q) < p(q')$ where q' are neighbor configurations of q and $q \neq q_f$. A random walk commences from configuration q for a fixed length (which could be randomly determined as well) to move the object through collision free configurations, potential values notwithstanding. Then a potential field guided search is performed to look for subsequently lower potential configurations q''. If $p(q'') < p(q)$, we have successfully escaped from local minimum configuration q. Otherwise, another random walk sequence commences from q.

Whereas such techniques are necessary for real life applications where only imperfect potential is available, there is a non–negligible cost associated with using them. Consider the components of a typical path as a result. Let a motion path be $\boldsymbol{t}$. It consists of two parts: one called the "down hill part" t_d that has configurations obtained from a search guided by the potential field; and the other "random walk part" $\boldsymbol{t_r}$ obtained from random walks that led to the escape from a local minimum. More specifically, $\boldsymbol{t}$ is an ordered set $\{s_i \mid i = \{d, r\}, s_d \subseteq \boldsymbol{t}_d, s_r \subseteq \boldsymbol{t}_r\}$. The lengths of $|\Sigma s_d|$ and $|\Sigma s_r|$ is a function of the application problem and the potential field techniques employed. If $|\Sigma s_d| = 0$, then the employed potential field technique was so in–effective as to have produced a path solely relying on random walk. On the other hand, if $|\Sigma s_r| = 0$, there is no local minimum encountered during the discovery of the path. In that case a path can be obtained through a straight forward steepest decent search. More realistically, a potential field will have local minima, and any s_d path seg-

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ment leading into a local minimum will have to be followed by an s_r segment in order to successfully generate a complete path to reach q_f. In our experience with applications reported in [4], an original motion path obtained with the above approach often has about the same order of magnitude configurations in t_r as in t_d.

The total cost to arrive at t is obviously the sum of total cost in generating t_r as well as t_d. While the indirect cost associated with obtaining t_r is a function of the given application, it is apparent that the less informed the potential in use, the larger the t_r portion of the final path. Hence it is desirable to reduce the length of or to eliminate t_r so that the total processing time approaches optimum (as defined in a local–minimum free potential field).

In [4] we showed a new application domain for motion planning techniques to play a key role to help designers of complicated mechanical systems to perform assemblability and maintainability studies in place of physical mock–up. There we showed by providing a constraint volume V_c, a path can be found efficiently. In constrained motion planning, a potential field is built by first propagating inside V_c, followed by expanding out on the surface of V_c through the rest of free space. A side effect of this approach is that while the potential inside the constraint volume is more informed as to where in general a moving object should go through, there will be more local minima as a result of imposing the constraint volume. In using the system reported in [4], we observed a large cost associated with using random walks to get out of those local minima. In this paper, we describe a new technique to overcome this problem by reducing the cost associated with random walk techniques. Some application examples are given to illustrate benefits achievable by the new technique. In the discussion section we analyze the characteristics of applications that could benefit from this new technique.

3. METHOD

In general, the force exerted onto a moving object is the integral effect of the potential over the whole object. A potential value over a configuration q does not result any movement in itself. Rather, it is the gradient of the potential values over a vicinity of q that creates a differential force which results in a motion from q. For instance, if we follow the steepest decent of a potential, the strongest gradient differential will be the direction along which the object will be moving under the influence of the field.

In reality, since it is too expensive to evaluate this integral over the whole 3D moving object during a search, only a subset of the object is evaluated against the field and their effects combined to approximate the integral. Given a 3D object with non–zero volume, the above differential force is indeed a composite effect of potential field values calculated over several *control points*. (A straight forward composition or arbitration function would be to take the sum of the potential differential at each control point to effect a *global* force, which is a 6d vector in c–space. Various compositions have been studied in the literature, with different results depending on the problems applied.) This effect results a motion vector pointing to the desired neighboring configuration q'. Typically for a given application, there exists a global minimum for a potential field over a control point which is located at the final configuration q_f. In other words, it is always desirable to follow the strongest negative gradient to pursue "down–hill" motion (hence the t_d notion used in the last section). However, when the gradient is positive in all directions from q, or we have a point vector as the result of composition, the object is said to be in a local minimum at q.

We observe that, given a fixed configuration for a given 3D object, the potential fields over different control points could have different gradient or motion effects, principal of which in question is the fact that when one control point is at a local minimum, as indicated by a positive gradient differential in all directions in c–space, other potential fields could still yield negative gradients, in which case when applied to the object these negative gradients would move the object (in the right direction) towards q_f.

We illustrate this particular observation in Figure 1. In the figure, there exists a channel through which a potential field would pull a point object through to the global minimum. Given $\boldsymbol{A}$, we see in Figure 1(b) that the potential field on control point $\boldsymbol{a}$ is pulling $\boldsymbol{A}$ through the fissure, resulting a local minimum (since the shape of $\boldsymbol{A}$ does not allow its passage between $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$). Notice however in Figure 1(c) at the same configuration the potential field on control point $\boldsymbol{b}$ effectively does not result a local minimum of $\boldsymbol{A}$ into the fissure. Even if $\boldsymbol{A}$ was in a local minimum led in by the potential field corresponding to control point $\boldsymbol{b}$, the "submerged" part of $\boldsymbol{A}$ would have been smaller than that led in via control point $\boldsymbol{a}$, thus translating into a smaller effort needed to jump or walk out of a shallower local minimum.

Another related observation is on the fact that typically potential fields are calculated on a grid. Some local minima can be made invisible to another potential calculated at a different resolution. Hence, when a local minimum is present to a control point, it may not be present to the same control point in another potential field computed at a different resolution. This is true in general to cases where fine details result in local minima, while at a coarser resolution the field is smoother.

Based on the observations we formulate the following structured method.

Let $\boldsymbol{p}$ be a collection of potential fields $\{p_1, p_2, \ldots p_i, \ldots p_n\}$ where p_i is a potential field defined in the 3D workspace. Let q be a configuration in an n–dimensional c–space. The key desired properties of $\boldsymbol{p}$ are as follows: 1) p_i and p_j can not be linearly dependent; i.e., $\forall\ q \subseteq \Re^n\ \exists\ i$ where $p_i(q)$ is not a local minimum. 2) p_i and p_j have the same global minimum at the goal configuration, i.e., $\forall i\ j,\ p_i(q_f) = p_j(q_f)$.

Further, let $\mathcal{H}$ be the mechanism that chooses p_i at one instance in time to guide the search. $\mathcal{H}$ switches p_i to p_{i+1} at

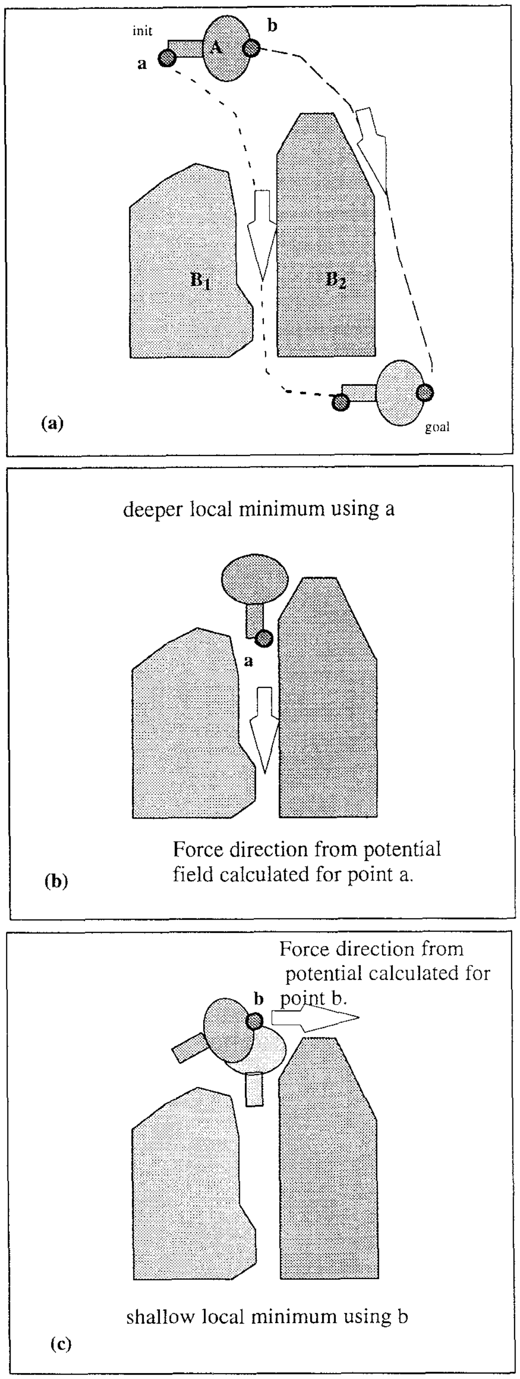

Figure 1. (a) shows two potential pulls. (b) shows *A* getting stuck in the fissure when the potential calculated on control point *a* is effecting a pull. (c) shows a different pulling force exerted by a potential on control point *b*. This figure illustrates the fact that even at the same configuration, a potential can effect different motion gradient on a 3D object.

$q \subseteq \Re^n$ iff $p_i(q)$ is a local minimum and $q \neq q_f$. If i+1>n, i+1=1.

With this mechanism, a new, multi–potential based search would proceed as follows.

Given q_0 as the start configuration, we use p_1 to guide the search of q_{1i} where i = 1, 2, 3,...k, where q_{11} is the configuration next to q_0 in the current path, q_{12} is next to q_{11}, and $p_1(q_{1k})$ is a local minimum. Then, $\mathcal{H}$ switches from using p_1 to using p_2, at q_{1k}, to find q_{2i} (i=1, ..., m) where $p_2(q_{2m})$ is a local minimum; and $\mathcal{H}$ switches to using p_3, etc. When finished, we have a path that is this ordered set of configurations $\boldsymbol{t} = (q_0, q_{11}, \ldots q_{1k}, q_{21}, \ldots q_{2m}, q_{31}, \ldots, q_f)$, where q_{ij} is the jth consecutive configuration discovered using potential field p_i, and q_f is the goal configuration. When this new mechanism succeeds in finding a path, we resorted to no random walks to obtain $\boldsymbol{t}$, or we have $\boldsymbol{t} = \boldsymbol{t}_d$, which is the optimal case in this context.

Note the potential field switching order can be a randomized selection or any other organized process, in addition to the circular form given above.

4. APPLICATION RESULTS

We use the following application to illustrate the effectiveness of the proposed approach. In Figure 2, we study the

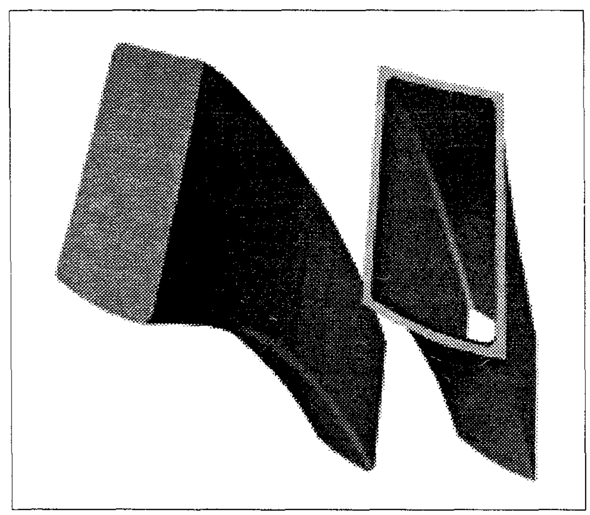

Figure 2. An insert and sleeve case.

feasibility of assembling an insert into a sleeve, both of which have non–trivial geometries. By using a path planning approach, we specified that the initial configuration for the insert is its installed position (inside the sleeve) and the goal configuration is a configuration where the insert is completely outside the sleeve along its major axis. Intuitively, one can see that the insert has to continuously rotate and translate to clear the inside wall of the sleeve. Given conventional distance function based potential field construction methods using disretized representation, there is not an obvious way to construct a field that will effect such fine, continuous motion. In our application, we used only one control point affixed to the head of the insert, and the resulting potential exerts a linear pulling force on the insert approximately along the major axis of the insert. This pulling force created a few local minima. We illustrate the effectiveness of the new multi–potential technique in a simplified 2D sketch in Figure 3.

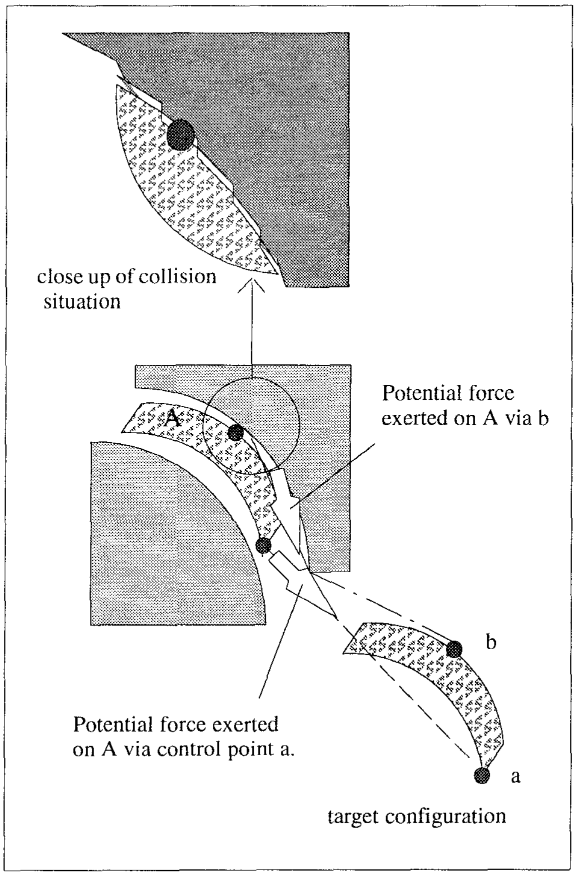

Figure 3. Different effects on A by using potential field calculated on different control points a, and b. Using a, A is stuck in the current configuration inside the assembly due to collision between the northeastern facing surface of A and the upper obstacle (as shown in the close–up). Using b, a downward motion will lead A closer to its target configuration q_f, until the concave features of A collide with the lower obstacle. Then a pull on b will move A away from this collision in the general direction towards q_f. By iterating this switching on the two control points or on their dependent potential fields, A can be moved out without resorting to random walk like local minimum handling techniques.

In the figure, an arched object (i.e. insert) is stuck in a local minimum when pulling a. Observe that the local minimum is a result of collision between the northeastern side of the insert against the upper obstacle. However, if the pulling force is from control point b, then there is still room for the insert to move, somewhat towards the 5 o'clock direction. One can see by constantly switching between the two control points in this case, the insert will move smoothly over the illustrated neighborhood. One can also see that at a single configuration, there must be one unique force or potential that will effect the largest amount of motion. Overall, switching between potentials to effect a different pull may be the ultimate solution to yield an optimal path without a dynamic arbitration function.

In fact, in a similar case where the complexity of the parts are on the order of 100K polygons for the sleeve, and 20K polygons for the insert, we tested the new technique to achieve a factor of 10 improvement in speed over the standard random walk technique. Furthermore, the resultant path has about 1000 steps, as opposed to a typical path of 4000 steps obtained with random walks.

5. DISCUSSIONS

The random walk segments $\mathbf{t}_r$ is perhaps the most irregular in their resulting motion smoothness. Initially when a path is obtained, an optimization (or smoothing) step is often necessary to make the motion steps more coherent. During this process, parts of the original path are replaced with straight line segments in c–space which do not result in collision. A cursory observation would conclude that for the most part, it is those t_r segments that will get replaced. Hence, the cost of creating them is not fully reimbursed. On this note, our technique does not incur such costs in the first place.

It is especially important to note that, given a well constructed potential field, local minima relate to "details" that a planner have to maneuver through to effectively steer the object clear of all obstacles while staying *on course*. Such a course could be visually seen for instance, in our previously reported constrained motion planning, as a constraint volume [8]. Within the constraint volume if a path exists, it should be along the potential field with the general direction outlined by the constraint volume. With this new random–walk free approach, we have observed that it is indeed a more efficient alternative technique to handle local minimum prone situations.

The effect of this new method seems to resemble what would be achieved through a one–step random walk. If one considers the fact that given $p_1(q)$ is a local minimum, then if $p_2(q)$ does not have the same value, using $\mathcal{K}$ effects a random walk "over the potential field on the same configuration q" to get out of local minimum q. The distance $| p_2(q) - p_1(q) |$ is actually the stride of this "random walk."

With this new technique, by alternating potential fields we achieve great performance improvements to generate a linear sequence of paths obtained from using different potential fields, each of which makes its most contribution in guiding the motion (to stop only when it cannot move any more – in a local minimum configuration). However, the new technique is not perfect.

First, motion thrashing may occur when effectively different potential fields actually result, from a given configuration q, a circular motion to come back to q after several moves. This is the result of local inconsistencies in potentials p_i which have different local minima over non–goal configurations. It is also because that there is no local coherence in where each field would pull the moving object near q. A solution to this problem is to resort to a set of controlled random walks to break a loop when detected.

Another problem lies in the fact that for applications where a potential field would be so ill informed that a real solution will have to be found through random walks or even backtracks, the new technique may delay the exploration of these distant areas in c–space. The amount of perturbation the new technique can achieve is a function of how different p_i are from each other, and how effective the switching mechanism is capable of responding to frequent switching which is an indication of inefficiency. If the perturbation is small, the new technique won't explore drastically different neighborhoods in c–space. If the only existing path is far away from the c–space vicinity the current partial path ends, the new technique will fail.

Applications that are indicative of such conditions include moving a part to the other side of a fence when the only one opening between the bars that is wide enough for passage is far from the opening over which the moving object is located initially. Figure 4 illustrates this situation in 2D in a birds' eye view. In the figure, A is over fence opening a, making an attempt to pass to the other side of the fence, and opening z is the only one passable. Note the potential is strongest in this case over opening a, which indicates that for A to make through z, various random walks have to be performed to transition A over to z. The solution to this problem may be in using a constraint volume where a constraint sphere [8] is placed over z to effect a potential that pulls A to over z first. Then the new technique provides a perturbation force to negotiate the passage of A through z.

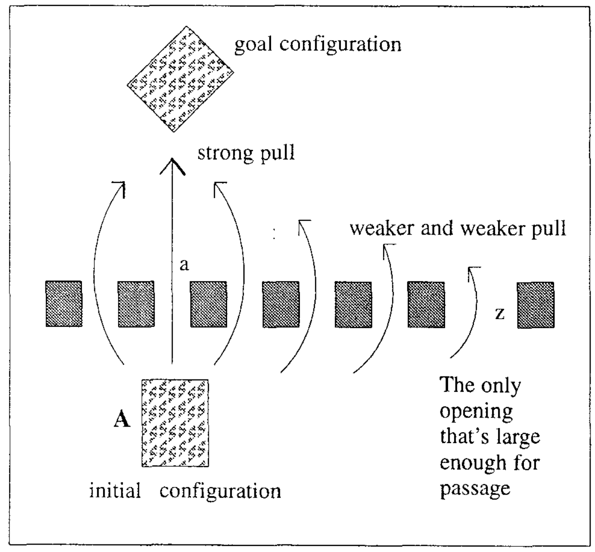

Figure 4. A fence obstacle and its potential field

6. CONCLUSIONS

We described a new technique to handle local minima in imperfect potential field based find–path applications. It is especially effective for constrained path planning because the potential field inside a constraint volume provides a good general direction, while the new technique presents an efficient perturbation technique to replace traditional random walk mechanisms in guiding a search along this general direction. Over 10–fold performance improvements have been achieved in applying the technique to real life applications that resemble characteristics similar to that shown in Figure 2.

We plan to extend our research into exploring a more intelligent switching mechanism so that other available information such as motion progress per potential can be used to expedite the search further.

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