Inductive Learning of Answer Set Programs v3.1.0

User Manual

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August 16, 2017

Contents

Abstract

ILASP[8] (Inductive Learning of Answer Set Programs) is a system for learning ASP (Answer Set Programming) programs from examples. It is capable of learning normal rules, choice rules and constraints through positive and negative examples of partial answer sets. ILASP can also learn preferences, represented as weak constraints in ASP by considering example ordered pairs of partial answer sets.

1 Background and Installation

Inductive Logic Programming (ILP)[12] is the field of research considering logic-based machine learning. The goal is to find an hypothesis which, together with some given background knowledge, entails a set of positive examples and does not entail any of a set of negative examples. Mostly, ILP research has addressed the problem of learning definite clauses intended for use in Prolog programs.

More recently, there has been work on learning under the stable model semantics[6]. In [14], the concepts of *brave* and *cautious* induction were introduced. The task of brave induction is to find an hypothesis H such that $B \cup H$ (the background knowledge augmented with H) has at least one answer set which satisfies the examples; cautious induction on the other hand requires that $B \cup H$ is has at least one answer set and that every answer set satisfies the examples.

Early algorithms such as [2, 13] solved brave induction tasks. In [7], we gave examples of hypotheses which could not be learned by brave or cautious induction alone and presented a new learning framework *Learning from Answer Sets* which is capable of expressing both brave and cautious induction in the same learning task.

Our algorithm, ILASP (Inductive Learning of Answer Set Programs) is sound and complete with respect to the optimal (shortest) Learning from Answer Sets tasks. In [9], we presented an extension to our Learning from Answer Sets framework, *Learning from Ordered Answer Sets* which is capable of learning preferences represented as *weak constraints*.

ILASP2, is sound and complete with respect to the optimal solutions of any Learning from Ordered Answer Sets task.

1.1 Answer Set Programming

This document should not be used as an introduction to Answer Set Programming (ASP). Good introductions to ASP are [1, 5, 4]. In this section, we introduce the subset of ASP which ILASP uses.

In this document, we consider a *literal* to be either an *atom*, the negation as failure of an atom (written `not a` for an atom `a`) or a comparison between two terms such as $\mathtt{X} > \mathtt{2}$ or $\mathtt{4} != \mathtt{Y}$.

A normal rule is written: $\mathtt{h :- b_1, \ldots, b_n}$ where h_1 is an atom and $b_1 \ldots b_n$ are literals. Such a normal rule is read as "if $b_1 \ldots b_n$ are all true, then h must also be true".

A *normal logic program* is a set of normal rules. The Herbrand Base of a program P (written HB_P) is the set of all ground atoms which can be constructed using predicate names, constant symbols and function symbols in P. The Herbrand Interpretations of P are the subsets of HB_P and each interpretation I is interpreted as assigning any atom in I to be true and any other atom to false.

To solve a normal program P for *answer sets*, it is first replaced with an equivalent ground program. An interpretation I is an *answer set* of a ground program P if and only if it is a minimal model of the *reduct* of P with respect to I. This reduct (written P^I) is construced in two steps: firstly, remove any rule in P which contains the negation of an atom in I; secondly, remove any remaining negative literals from the remaining rules. For more details, see any of the references mentioned at the beginning of this section.

A (hard) constraint, is essentially a normal rule without a head. This empty head is often read as *false*. The meaning of a constraint $\mathtt{:- b_1, \ldots, b_n}$ is that if $b_1 \ldots b_n$ are satisfied by an interpretation, then it cannot be an answer set.

A choice rule is written $\mathtt{l\{h_1; \ldots; h_m\}u :- b_1, \ldots, b_n}$. where l and u are integers such that $0 \leq l \leq u \leq n$. Informally, the meaning of such a rule is that if the body is satisfied then between l and u of the atoms $h_1, \ldots, h_n$ must be satisfied. A simplified version of the reduct for programs containing normal rules, choice rules and constraints can be found at [10].

The final type of rule that we allow in ILASP is called a weak constraint. Weak constraints do not effect the answer sets of a program P, but they specify an ordering over the answer sets of P saying which answer sets are preferred to others. ASP can solve programs for optimal answer sets (answer sets which are not less preferred than any other answer set).

Example 1.1. Consider the program:

```
coin(1..3). % Note that this is a shorthand for coin(1). coin(2). coin(3).

1 { heads(C); tails(C) } 1 :- coin(C).

:~ heads(C).[1@2, C]
:~ tails(C).[1@1, C]
```

The choice rule in this program says that every coin is either *heads* or *tails*. This naturally leads to 8 answer sets in which the 3 coins take every combination of values.

The first weak constraint has priority 2 (read from the "@2"), whereas the second has priority 1. This means that the first weak constraint is more important and we solve this first. By this, we mean we select the answer sets which are best according to this weak constraint. If there is more than one such answer set, then we compare these remaining answer sets using the lower priority weak constraint.

The first weak constraint says that we must pay a penalty of 1 for every coin that takes the value heads. This means that if we have 0 coins which are heads we pay 0, if we have 1 head we pay 1 etc. This means that the only optimal answer set is the one in which all coins are tails.

The list terms appearing at the end of a weak constraint (after the priority level) are there to specify which constraints should be considered unique; for example, if we ground the first weak constraint in the previous program we get:

```
coin(1..3). % Note that this is a shorthand for coin(1). coin(2). coin(3).
:~ heads(1).[1@2, 1]
:~ heads(2).[1@2, 2]
:~ heads(3).[1@2, 3]
```

This means that each weak constraint is considered unique, and so we must pay the penalty for each weak constraint whose body is satisfied.

Consider instead the similar program:

```
coin(1..3). % Note that this is a shorthand for coin(1). coin(2). coin(3).

1 { heads(C); tails(C) } 1 :- coin(C).

:~ heads(C).[1@2]
:~ tails(C).[1@1, C]
```

The answer sets of this program remain the same, but now when we ground the first weak constraint we get:

```
coin(1..3). % Note that this is a shorthand for coin(1). coin(2). coin(3).
:~ heads(1).[1@2]
:~ heads(2).[1@2]
:~ heads(3).[1@2]
```

As "`[1@2]`" occurs for all three weak constraints, we only have to pay the penalty once if we have any heads. This means that we pay penalty 0 if we have no heads, and penalty 1 if we have 1, 2 or 3 heads.

More formally, a weak constraint is written $:\sim \texttt{b}_1, \ldots, \texttt{b}_\texttt{n}.[\texttt{w@l}, \texttt{t}_1, \ldots, \texttt{t}_\texttt{m}]$ where $b_1, \ldots, b_n$ are literals, w is either a variable or an integer called the weight, l is an integer called the level and $t_1, \ldots, t_m$ are terms. A ground weak constraint of this form has the meaning that if $b_1, \ldots, b_n$ are satisfied by an answer set A then we must pay the penalty w. The terms t_i are to specify which of the ground weak constraints should be considered distinct (if two weak constraints have the same w, l and $t_1, \ldots, t_n$ we will only pay the penalty once, even if both bodies are satisfied). The aim is to find an answer set which pays the minimum penalty. The idea of the levels are that we should solve the higher levels first (considering them as a higher priority). Only if two answer sets pay equal penalties for all levels above l do we consider the weak constraints of level l.

1.2 Installation

1.2.1 Linux

Note that although in theory ILASP should work on any Linux distribution, we have only tested it on Ubuntu 16.04.

Clingo. [3] ILASP depends on clingo version 4.3.0. It is important that exactly this version[1] is used due to differences in clingo's built in scripting language in later versions. At the time of writing clingo 4.3.0 can be downloaded from `http://downloads.sourceforge.net/project/potassco/clingo/4.3.0/clingo-4.3.0-x86_64-linux.tar.gz`. The clingo executable should then be copied somewhere in your `PATH`. Our recommended approach is to copy the executable into the `/usr/local/bin/` directory.

It is now possible to run ILASP with newer versions of clingo. For details, see Section 1.3.1.

Poco. Older versions of ILASP use the poco library. This library can be installed in Ubuntu using the command `sudo apt-get install libpoco-dev`. This is not necessary from ILASP 3.0.0 onwards.

The current version of ILASP for linux can be obtained from `http://sourceforge.net/projects/spikeimperial/files/ILASP/`. It is recommended that you again copy the executable to `/usr/local/bin/`.

1.2.2 Mac OSX

ILASP has been built and tested on OSX 10.12 (Sierra).

Clingo. [3] ILASP depends on clingo version 4.3.0. It is important that exactly this version[1] is used due to differences in clingo's built in scripting language in later versions. At the time of writing clingo 4.3.0 can be downloaded from `http://downloads.sourceforge.net/project/potassco/clingo/4.3.0/clingo-4.3.0-macos-10.9.tar.gz`. The clingo executable should then be copied somewhere in your `PATH`. Our recommended approach is to copy the executable into the `/usr/local/bin/` directory.

It is now possible to run ILASP with newer versions of clingo. For details, see Section 1.3.1.

Poco. Older versions of ILASP use the poco library. This library can either be downloaded from `http://pocoproject.org/download/` or installed in OSX using the brew[2] package manager (`brew install poco`). This is not necessary from ILASP 3.0.0 onwards.

The current version of ILASP for OSX can be obtained from `http://sourceforge.net/projects/spikeimperial/files/ILASP/`. It is recommended that you again copy the executable to `/usr/local/bin/`.

[1] Note that if you decide to compile clingo 4.3.0 from source, it is *vital* that you compile it with Lua support

[2] for details of how to install brew see `http://brew.sh`.

1.3 How to run ILASP

Once you have installed ILASP, you should be able to run the example tasks available at `http://sourceforge.net/projects/spikeimperial/files/ILASP/`. Assuming you have an ILASP task stored at the location "`path_to_task`", you can use ILASP to solve this task for an optimal solution by using a command of the following form:

```
ILASP --version=[1|2|2i|3] [options] path_to_task
```

Note that from ILASP version 3.1.0 onwards, it is required to specify the version of the ILASP algorithm that you wish to run. Different versions of ILASP are suited for different types problems, so it is recommended that you experiment with the various versions. A summary of the strengths and weaknesses of the different versions is given below.

- **ILASP1** is available only for completeness, and is unlikely to outperform ILASP2.
- **ILASP2** is likely to perform best on tasks with few examples. It does not scale with large numbers of examples.
- **ILASP2i** is an iterative version of the ILASP2 algorithm, which is designed to scale with the number of examples. ILASP2i does not scale very well with large numbers of noisy examples.
- **ILASP3** is targeted at learning from large numbers of noisy examples. It does a lot of work in processing the examples individually, rather than all at once. In some cases, the computational cost of this processing can sometimes be more expensive than the time it saves, especially in tasks with large hypothesis spaces.

1.3.1 Options

ILASP has several options which can be given before or after the path to the task which is to be solved. These are summarised by table 1.

Option	Description
`-s`	Print the search space for the task.
`-q`	Solve the task as normal, but only print the hypothesis.
`-d`	Solve the task as normal, and print debugging information throughout the computation (this is only applicable in 2i and 3).
`-t`	In ILASP1 and ILASP2, this prints the ILASP ASP encoding for the task and exits. In ILASP2i, it causes each of the intermediate ASP encodings to be written to a file (the name of which is displayed in the debugging output).
`-pt`	Solve the task, first pre-translating away the contexts (should be used together with version 2).
`-ni`	(for use with version 3) Solve the task with no implication check.
`-np`	(for use with version 3) Solve the task with no propagation.
`--clingo "path_to_clingo clingo_options"`	use the specified path to clingo and options to clingo. Note that some options will interfere with the output ILASP is expecting, and are not safe to use. This should really only be used to change the heuristics and configuration parameters for clingo.
`--clingo4.5`	This tells ILASP that the clingo executable it is using expects the clingo 4.5 input format (by default, ILASP expects clingo 4.3).
`--clingo5`	This tells ILASP that the clingo executable it is using expects the clingo 5 input format.

Table 1: A summary of the main options which can be passed to ILASP.

2 Learning from Answer Sets

In [7] we specified a new Inductive Logic Programming task for learning ASP programs from examples of *partial interpretations*.

A partial interpretation E is a pair of sets of atoms E^{inc} and E^{exc}, called the *inclusions* and *exclusions* of E. We write E as $\langle E^{inc}, E^{exc}\rangle$. An answer set A is said to *extend* E if it contains all of the inclusions ($E^{inc} \subseteq A$) and none of the exclusions ($E^{exc} \cap A = \emptyset$)).

In *Learning from Answer Sets* (ILP_{LAS}), given an ASP program B called the background knowledge, a set of ASP rules S called the search space and two sets of partial interpretations E^+ and E^- called the positive and negative examples respectively, the goal is to find another program H called an hypothesis such that:

1. H is composed of the rules in S ($H \subseteq S$).
2. Each positive example is extended by at least one answer set of $B \cup H$ (this can be a different answer set for each positive example).
3. No negative example is extended by any answer set of $B \cup H$.

This task was later upgraded to support *context-dependent* examples, which allow extra (specific) background knowledge to be given with each example. Context-dependent examples are discussed in Section 5.

2.1 Specifying simple learning tasks in ILASP

A positive example $\langle\{e_1^{inc}, \ldots e_m^{inc}\}, \{e_1^{exc}, \ldots, e_n^{exc}\}\rangle$ can be specified in ILASP as the statement: `#pos(example_name,` $\{\mathtt{e}_1^{\mathtt{inc}}, \ldots, \mathtt{e}_\mathtt{m}^{\mathtt{inc}}\}, \{\mathtt{e}_1^{\mathtt{exc}}, \ldots, \mathtt{e}_\mathtt{n}^{\mathtt{exc}}\}$`).` where `example_name` is a unique identifier for the example. Similarly a negative example can be specified using the predicate `#neg`.

A rule r in the search space with length l is specified as $l \sim r$ and finally, rules in the background knowledge are specified as normal. The length of rules in the search space is used when determining which hypotheses are optimal.

Example 2.1. The ILASP task:

```
p :- not q.
q :- not p.

% At least one answer set should contain but not
% contain r.
#pos(p1, {q}, {r}).

% At least one answer set should contain both q
% and r.
#pos(p2, {q, r}, {}).

% At least one answer set should contain p.
#pos(p3, {p}, {}).

% No answer set should contain both p and r.
#neg(n1, {p, r}, {}).

1 ~ r.
1 ~ s.
2 ~ r :- not p.
2 ~ r :- p.
2 ~ s :- not p.
2 ~ s :- p.
1 ~ :- not p.
1 ~ :- p.
2 ~ s :- not r.
2 ~ s :- r.
1 ~ :- not r.
1 ~ :- r.
2 ~ r :- not s.
2 ~ r :- s.
1 ~ :- not s.
1 ~ :- s.
3 ~ s :- not p, not r.
3 ~ s :- p, not r.
3 ~ s :- not p, r.
3 ~ s :- p, r.
2 ~ :- not p, not r.
2 ~ :- p, not r.
2 ~ :- not p, r.
2 ~ :- p, r.
3 ~ r :- not p, not s.
3 ~ r :- p, not s.
3 ~ r :- not p, s.
3 ~ r :- p, s.
2 ~ :- not p, not s.
2 ~ :- p, not s.
2 ~ :- not p, s.
2 ~ :- p, s.
2 ~ :- not r, not s.
2 ~ :- r, not s.
2 ~ :- not r, s.
```

```
2 ~ :- r, s.
3 ~ :- not p, not r, not s.
3 ~ :- p, not r, not s.
3 ~ :- not p, r, not s.
3 ~ :- p, r, not s.
3 ~ :- not p, not r, s.
3 ~ :- p, not r, s.
3 ~ :- not p, r, s.
3 ~ :- p, r, s.
```

when solved with ILASP gives the output:

```
r :- not p, not s.
s :- not r.

Pre-processing  : 0.003s
Solve time      : 0.018s
Total           : 0.021s
```

In this case, the answer sets of $B \cup H$ are $\{\mathtt{p}, \mathtt{s}\}$, $\{\mathtt{q}, \mathtt{s}\}$ and $\{\mathtt{q}, \mathtt{r}\}$. Note that these three answer sets extend the three positive examples, and no answer set extends the negative example.

2.2 Language Bias

The ILASP procedure depends on a *search space* of possible hypotheses in which it should search for inductive solutions. Although in ILASP the search space can be given in full as in the previous section, the conventional way of specifying this search space is to give a *language bias* specified by *mode declarations* (this is sometimes called a *mode bias*).

A placeholder is a term `var(t)` or `const(t)` for some constant term `t`. The idea is that these can be replaced by any variable or constant (respectively) of *type* t.

Each constant `c` which is allowed to replace a `const` term of type `t` should be specified as `#constant(t, c)`

A mode declaration is a ground atom whose arguments can be placeholders. An atom a is *compatible* with a mode declaration m if each of the placeholder constants and placeholder variables in m has been replaced by a constant or variable of the correct type (the user does not have to list the variables which can be used, but ILASP will ensure that no variable occurs twice in the same rule with a different type).

There are three types of mode declaration: `modeh` the *normal head* declarations; `modeha`, the `aggregate head` declarations; `modeb`, the body declarations; and `modeo`, the optimisation body declarations. Each mode declaration can also be specified with a *recall*, which is an integer specifying the maximum number of times that mode declaration can be used in each rule.

A user can specify a mode declaration stating that the predicate p with arity 1 can be used in each rule up to two times with a single variable of type `type1`, for example, as `#modeb(2, p(var(type1)))`.

The maximum number of variables in any rule can be specified using the predicate #`maxv`; for example `#maxv(2)` states that at most 2 variables can be used in each rule of the hypothesis. Similarly, #`max_penalty` specifies an upper bound on the size of the hypothesis returned. By default, this is 15. Lower values for this are likely to increase the speed of computation, but in some cases larger bounds are needed (as in the sudoku example in the next section).

Example 2.2. The language bias:

```
#modeha(r(var(t1), const(t2)))
#modeh(p)
#modeb(1, p)
#modeb(2, q(var(t1)))
#constant(t2, c1)
#constant(t2, c2)
#maxv(2).
```

Leads to the search space:

```
:- q(V1).
:- p.
:- not p.
:- q(V1), p.
:- q(V1), not p.

p.
p :- q(V1).

0 { r(V1, c1) } 1 :- q(V1).
0 { r(V1, c1) } 1 :- q(V1), p.
0 { r(V1, c1) } 1 :- q(V1), not p.

0 { r(V1, c2) } 1 :- q(V1).
0 { r(V1, c2) } 1 :- q(V1), q(V2).
0 { r(V1, c2) } 1 :- q(V1), q(V2), p.
0 { r(V1, c2) } 1 :- q(V1), p.
0 { r(V1, c2) } 1 :- q(V1), q(V2), not p.
0 { r(V1, c2) } 1 :- q(V1), not p.

0 { r(V1, c1); r(V1, c2) } 1 :- q(V1).
0 { r(V1, c1); r(V1, c2) } 1 :- q(V1), p.
0 { r(V1, c1); r(V1, c2) } 1 :- q(V1), not p.
1 { r(V1, c1); r(V1, c2) } 1 :- q(V1).
1 { r(V1, c1); r(V1, c2) } 1 :- q(V1), p.
1 { r(V1, c1); r(V1, c2) } 1 :- q(V1), not p.
1 { r(V1, c1); r(V1, c2) } 2 :- q(V1).
1 { r(V1, c1); r(V1, c2) } 2 :- q(V1), p.
1 { r(V1, c1); r(V1, c2) } 2 :- q(V1), not p.

0 { r(V1, c1); r(V2, c1) } 1 :- q(V1), q(V2).
0 { r(V1, c1); r(V2, c1) } 1 :- q(V1), q(V2), p.
0 { r(V1, c1); r(V2, c1) } 1 :- q(V1), q(V2), not p.
1 { r(V1, c1); r(V2, c1) } 1 :- q(V1), q(V2).
1 { r(V1, c1); r(V2, c1) } 1 :- q(V1), q(V2), p.
1 { r(V1, c1); r(V2, c1) } 1 :- q(V1), q(V2), not p.
1 { r(V1, c1); r(V2, c1) } 2 :- q(V1), q(V2).
1 { r(V1, c1); r(V2, c1) } 2 :- q(V1), q(V2), p.
1 { r(V1, c1); r(V2, c1) } 2 :- q(V1), q(V2), not p.

0 { r(V1, c1); r(V2, c2) } 1 :- q(V1), q(V2).
0 { r(V1, c1); r(V2, c2) } 1 :- q(V1), q(V2), p.
0 { r(V1, c1); r(V2, c2) } 1 :- q(V1), q(V2), not p.
1 { r(V1, c1); r(V2, c2) } 1 :- q(V1), q(V2).
1 { r(V1, c1); r(V2, c2) } 1 :- q(V1), q(V2), p.
1 { r(V1, c1); r(V2, c2) } 1 :- q(V1), q(V2), not p.
1 { r(V1, c1); r(V2, c2) } 2 :- q(V1), q(V2).
1 { r(V1, c1); r(V2, c2) } 2 :- q(V1), q(V2), p.
1 { r(V1, c1); r(V2, c2) } 2 :- q(V1), q(V2), not p.

0 { r(V1, c2); r(V2, c2) } 1 :- q(V1), q(V2).
0 { r(V1, c2); r(V2, c2) } 1 :- q(V1), q(V2), p.
0 { r(V1, c2); r(V2, c2) } 1 :- q(V1), q(V2), not p.
1 { r(V1, c2); r(V2, c2) } 1 :- q(V1), q(V2).
1 { r(V1, c2); r(V2, c2) } 1 :- q(V1), q(V2), p.
1 { r(V1, c2); r(V2, c2) } 1 :- q(V1), q(V2), not p.
1 { r(V1, c2); r(V2, c2) } 2 :- q(V1), q(V2).
1 { r(V1, c2); r(V2, c2) } 2 :- q(V1), q(V2), p.
1 { r(V1, c2); r(V2, c2) } 2 :- q(V1), q(V2), not p.
```

Note that ILASP will try to omit rules which will never appear in an optimal inductive solution; for example, as `p :- q(V1), q(V2).` is satisfied if and only if `p :- q(V1).` is satisfied, the former will never be part of an optimal solution, and so is not generated.

For details of how we assign lengths to rules in the search space, see section 6.

2.2.1 Extra options for mode declarations

A final argument may be given to any mode declarations, which is a tuple to restrict the use of this mode declaration. The idea is that this can be used to further steer the search away from hypotheses which are uninteresting and thus improve the speed of the search.

This final argument is a tuple which may contain the any of the constants: `anti_reflexive`, `symmetric`, `positive`. The meaning of these constants is as follows:

1. `anti_reflexive` is intended for use on predicates of arity 2. It means that the predicate should not be generated with 1 variable repeated for both arguments; for example, `p(X,X)` is not compatible with `#modeh(p(var(t),var(t)),(anti_reflexive))`.
2. `symmetric` is intended for use on predicates of arity 2. It means that, for example, given the mode declaration `#modeh(p(var(t),var(t)),(symmetric))`, rules containing `p(X, Y)` are considered equivalent to the same rule replaced with `p(Y, X)` and, as such, only one is generated.
3. `positive` means that the negation as failure of the predicate in the mode declaration should not be generated.

Other options, aimed mainly at choice rules (as it is very easy to generate a search space with a massive number of choice rules) are `#disallow_multiple_head_variables`, which prevents more than one variable being used in the head of a rule; `#minh` and `#maxh` which are predicates of arity 1 specifying the minimum and maximum number of literals which can occur in the head of a choice rule.

2.3 Example Tasks

In this section, we give example learning tasks encoded in ILASP. The first shows how ILASP can be used to learn the rules of sudoku. Further examples may follow in later editions of this manual.

2.3.1 Sudoku

Consider the following learning task whose background knowledge contains the definitions of what it means to be a cell, and for two cells to be in the same row, column and block:

```
cell((1..4,1..4)).

block((X, Y), tl) :- cell((X, Y)), X < 3, Y < 3.
block((X, Y), tr) :- cell((X, Y)), X > 2, Y < 3.
block((X, Y), bl) :- cell((X, Y)), X < 3, Y > 2.
block((X, Y), br) :- cell((X, Y)), X > 2, Y > 2.

same_row((X1,Y),(X2,Y)) :- X1 != X2, cell((X1,Y)), cell((X2, Y)).
same_col((X,Y1),(X,Y2)) :- Y1 != Y2, cell((X,Y1)), cell((X, Y2)).
same_block(C1,C2) :- block(C1, B), block(C2, B), C1 != C2.

#pos(a, {
  value((1, 1), 1),           % This positive examples says that there should be at
  value((1, 2), 2),           % least one answer set of B union H which represents a
  value((1, 3), 3),           % board extending the partial board:
  value((1, 4), 4),           %
  value((2, 3), 2)            %         #-------#-------#
}, {                          %         | 1 : 2 | 3 : 4 |
  value((1, 1), 2),           %         | - + - | - + - |  such that the first cell
  value((1, 1), 3),           %         |   :   | 2 :   |  does not also contain a
  value((1, 1), 4)            %         #-------#-------#  value other than 1.
}).                           %         |   :   |   :   |
                              %         | - + - | - + - |
                              %         |   :   |   :   |
                              %         #-------#-------#


#neg(b, { value((1, 1), 1), value((1, 3), 1) },{}).
#neg(c, { value((1, 1), 1), value((3, 1), 1) },{}).
#neg(d, { value((1, 1), 1), value((2, 2), 1) },{}).

% The negative examples say that there should be no answer set corresponding to a
% board extending any of the partial boards:
%
%     #-------#-------#      #-------#-------#      #-------#-------#
%     | 1 :   | 1 :   |      | 1 :   |   :   |      | 1 :   |   :   |
%     | - + - | - + - |      | - + - | - + - |      | - + - | - + - |
%     |   :   |   :   |      |   :   |   :   |      |   : 1 |   :   |
%     #-------#-------#      #-------#-------#      #-------#-------#
%     |   :   |   :   |      | 1 :   |   :   |      |   :   |   :   |
%     | - + - | - + - |      | - + - | - + - |      | - + - | - + - |
%     |   :   |   :   |      |   :   |   :   |      |   :   |   :   |
%     #-------#-------#      #-------#-------#      #-------#-------#

#modeha(value(var(cell),const(number))).

#modeb(2,value(var(cell),var(val))).
#modeb(1,same_row(var(cell), var(cell)), (anti_reflexive)).
#modeb(1,same_block(var(cell), var(cell)), (anti_reflexive)).
#modeb(1,same_col(var(cell), var(cell)), (anti_reflexive)).
```

```
#modeb(1,cell(var(cell))).

#constant(number, 1).
#constant(number, 2).
#constant(number, 3).
#constant(number, 4).

#minhl(4).
#maxhl(4).
#disallow_multiple_head_variables.
#max_penalty(30).
```

Calling ILASP on this task will give an output similar to:

```
:- value(V0,V1), value(V2,V1), same_col(V0, V2).
:- value(V0,V1), value(V2,V1), same_block(V0, V2).
:- value(V0,V1), value(V2,V1), same_row(V2, V0).
1 {value(V0,4); value(V0,3); value(V0,2); value(V0,1) } 1 :- cell(V0).

Pre-processing  : 0.028s
Solve time      : 0.91s
Total           : 0.938s
```

This hypothesis represents the rules of a 4×4 version of the popular game of sudoku. The choice rule says that every cell takes exactly one value between 1 and 4 and the three constraints rule out the 3 cases of illegal board where values occur more than once in the same column, block or row.

3 Learning Weak Constraints using ILASP

In [9], we defined a new task, *Learning from Ordered Answer Sets* which extends Learning from Answer Sets with the capability of learning weak constraints. As positive and negative examples do not incentivise learning weak constraints, we introduced a new notion of ordering examples. These come in two kinds, *brave* and *cautious*.

Definition 1. An *ordering example* is a tuple $o = \langle e_1, e_2 \rangle$ where e_1 and e_2 are partial interpretations. An ASP program P *bravely respects* o if and only if $\exists A_1, A_2 \in AS(P)$ such that A_1 extends e_1, A_2 extends e_2 and $A_1 \succ_P A_2$. P *cautiously respects* o if and only if for each $A_1, A_2 \in AS(P)$ such that A_1 extends e_1 and A_2 extends e_2, it is the case that $A_1 \succ_P A_2$.

The partial interpretations in ordering examples must be positive examples. Given two positive examples e_1 and e_2 with identifiers `id_1` and `id_2`, the ordering example $\langle e_1, e_2 \rangle$ can be expressed as `#brave_ordering(order_name,` $\texttt{id}_1, \texttt{id}_2$`)` or `#cautious_ordering(order_name,` $\texttt{id}_1, \texttt{id}_2$`)` where `order_name` is an optional identifier for the ordering example.

Example 3.1. Consider a learning task with positive examples `#pos(p1, {p}, {})` and `#pos(p2, {}, {p})` and a single ordering example `#brave_ordering(o1, p1, p2)`. ILASP will search for the shortest hypothesis H such that $B \cup H$ has answer sets to cover each positive example, and there is at least one answer set which contains `p` which is more optimal than at least one answer set which does not contain `p`.

If we consider the same positive examples, but now with the ordering example `#pos(p2, {}, {p})`, then ILASP will search for an hypothesis H such that $B \cup H$ has answer sets to cover each positive example, and every answer set which contains `p` is more optimal than every answer set which does not contain `p`.

We now define the notion of *Learning from Ordered Answer Sets* by extending Learning from Answer Sets with ordering examples.

Definition 2. A *Learning from Ordered Answer Sets* task is a tuple $T = \langle B, S_M, E^+, E^-, O^b, O^c \rangle$ where B is as ASP program, called the background knowledge, S_M is the search space defined by a mode bias with ordering M, E^+ and E^- are sets of partial interpretations called, respectively, positive and negative examples, and O^b and O^c are both sets of ordering examples over E^+. An hypothesis H is an *inductive solution* of T (written $H \in ILP_{LOAS}(T)$) if and only if:

1. $H' \in ILP_{LAS}(\langle B, S_M^{LAS}, E^+, E^- \rangle)$[3]
2. $\forall o \in O^b$ $B \cup H$ bravely respects o
3. $\forall o \in O^c$ $B \cup H$ cautiously respects o

Example 3.2. Consider the learning task:

```
animal(cat).   animal(dog).   animal(fish).   eats(cat, fish).   eats(dog, cat).
0 { own(A) } 1 :- animal(A).

#pos(p1, { own(fish) }, { own(cat) }).          #pos(p2, { own(cat) }, { own(dog), own(fish)}).
#pos(p3, { own(dog), own(cat), own(fish) }, {}).     #pos(p4, { own(dog)}, { own(fish) }).
#pos(best, { own(dog), own(fish) }, { own(cat) }).
                                                  % These ordering examples all use an example "best" in
                                                  % which a dog and a fish is owned and a cat is not owned.
                                                  %
#brave_ordering(best, p1).                        % "best" is better than some situations where we
                                                  % own a fish but no cat.
#brave_ordering(best, p2).                        % "best" is better than some situations where we
                                                  % own a cat but no fish or dog.
#brave_ordering(best, p3).                        % "best" is better than some situations where we
                                                  % own a fish, dog and cat.
#cautious_ordering(best, p4).                     % "best" is better than every situation in which
                                                  % we own a dog but no fish.
```

[3] S_M^{LAS} is the subset of S_M which are not weak constraints.

```
1 ~ :~ eats(V0, V1).[1@1, 1, V0, V1]
1 ~ :~ eats(V0, V1).[1@2, 2, V0, V1]
1 ~ :~ eats(V0, V1).[-1@1, 3, V0, V1]
1 ~ :~ eats(V0, V1).[-1@2, 4, V0, V1]
1 ~ :~ own(V0).[1@1, 5, V0]
1 ~ :~ own(V0).[1@2, 6, V0]
1 ~ :~ own(V0).[-1@1, 7, V0]
1 ~ :~ own(V0).[-1@2, 8, V0]
2 ~ :~ eats(V0, V1), not own(V0).[1@1, 9, V0, V1]
2 ~ :~ eats(V0, V1), not own(V0).[1@2, 10, V0, V1]
2 ~ :~ eats(V0, V1), not own(V0).[-1@1, 11, V0, V1]
2 ~ :~ eats(V0, V1), not own(V0).[-1@2, 12, V0, V1]
2 ~ :~ eats(V0, V1), not own(V1).[1@1, 13, V0, V1]
2 ~ :~ eats(V0, V1), not own(V1).[1@2, 14, V0, V1]
2 ~ :~ eats(V0, V1), not own(V1).[-1@1, 15, V0, V1]
2 ~ :~ eats(V0, V1), not own(V1).[-1@2, 16, V0, V1]
2 ~ :~ eats(V0, V1), own(V0).[1@1, 17, V0, V1]
2 ~ :~ eats(V0, V1), own(V0).[1@2, 18, V0, V1]
2 ~ :~ eats(V0, V1), own(V0).[-1@1, 19, V0, V1]
2 ~ :~ eats(V0, V1), own(V0).[-1@2, 20, V0, V1]
2 ~ :~ eats(V0, V1), own(V1).[1@1, 21, V0, V1]
2 ~ :~ eats(V0, V1), own(V1).[1@2, 22, V0, V1]
2 ~ :~ eats(V0, V1), own(V1).[-1@1, 23, V0, V1]
2 ~ :~ eats(V0, V1), own(V1).[-1@2, 24, V0, V1]
2 ~ :~ own(V0), own(V1).[1@1, 25, V0, V1]
2 ~ :~ own(V0), own(V1).[1@2, 26, V0, V1]
2 ~ :~ own(V0), own(V1).[-1@1, 27, V0, V1]
2 ~ :~ own(V0), own(V1).[-1@2, 28, V0, V1]
3 ~ :~ eats(V0, V1), not own(V0),
       not own(V1).[1@1, 29, V0, V1]
3 ~ :~ eats(V0, V1), not own(V0),
       not own(V1).[1@2, 30, V0, V1]
3 ~ :~ eats(V0, V1), not own(V0),
       not own(V1).[-1@1, 31, V0, V1]
3 ~ :~ eats(V0, V1), not own(V0),
       not own(V1).[-1@2, 32, V0, V1]
3 ~ :~ eats(V0, V1), own(V0),
       not own(V1).[1@1, 33, V0, V1]
3 ~ :~ eats(V0, V1), own(V0),
       not own(V1).[1@2, 34, V0, V1]
3 ~ :~ eats(V0, V1), own(V0),
       not own(V1).[-1@1, 35, V0, V1]
3 ~ :~ eats(V0, V1), own(V0),
       not own(V1).[-1@2, 36, V0, V1]
3 ~ :~ not eats(V0, V1), own(V0),
       own(V1).[1@1, 37, V0, V1]
3 ~ :~ not eats(V0, V1), own(V0),
       own(V1).[1@2, 38, V0, V1]
3 ~ :~ not eats(V0, V1), own(V0),
       own(V1).[-1@1, 39, V0, V1]
3 ~ :~ not eats(V0, V1), own(V0),
       own(V1).[-1@2, 40, V0, V1]
3 ~ :~ not eats(V1, V0), own(V0),
       own(V1).[1@1, 41, V0, V1]
3 ~ :~ not eats(V1, V0), own(V0),
       own(V1).[1@2, 42, V0, V1]
3 ~ :~ not eats(V1, V0), own(V0),
       own(V1).[-1@1, 43, V0, V1]
3 ~ :~ not eats(V1, V0), own(V0),
       own(V1).[-1@2, 44, V0, V1]
3 ~ :~ eats(V0, V1), own(V0),
       own(V1).[1@1, 45, V0, V1]
3 ~ :~ eats(V0, V1), own(V0),
       own(V1).[1@2, 46, V0, V1]
3 ~ :~ eats(V0, V1), own(V0),
       own(V1).[-1@1, 47, V0, V1]
3 ~ :~ eats(V0, V1), own(V0),
       own(V1).[-1@2, 48, V0, V1]
```

ILASP will produce the output:

```
:~ eats(V0, V1), own(V0), own(V1).[1@2, 1, V0, V1]
:~ own(V0).[-1@1, 2, V0]

Pre-processing  : 0.004s
Solve time      : 0.079s
Total           : 0.083s
```

This hypothesis means that the top priority is to not own animals which eat each other. Once this has been optimised, the next priority is to own as many animals as possible (note that the negative weight corresponds to maximising rather than minimising the number of times the body of this constraint is satisfied).

3.1 Language Bias for Weak Constraints

As before, it is possible in ILASP to simply give a search space containing weak constraints in full; however, it is more conventional, and more convenient to express a search space using a mode bias. There are tree additional concepts required to do this: `#modeo` behaves exactly like `#modeb`, but expresses what is allowed to appear in the body of a weak constraint; `#maxp` is used much like `#maxv` to express the number of priority levels allowed to be used in the hypothesis (ILASP will use 1 to n, where n is the value given for `#maxp`); and finally, `#weight` is an arity one predicate which can take as an argument, either an integer or a type which is used in the variables of some `#modeo`.

In this version of ILASP, it is not possible to specify which terms should appear at the end of a weak constraint. ILASP will generate weak constraints which contain the full set of terms in the body of the weak constraint and one additional unique term identifying the rule. If you need more control over this, then you can still input the weak constraints manually with your own terms.

Example 3.3. The language bias:

```
#modeo(assigned(var(day), var(time)), (positive)).
#modeo(1, type(var(day), var(time), const(course)), (positive)).
#modeo(1, var(day) != var(day)).
#constant(course, c2).
#weight(1).
#maxv(4).
#maxp(2).
```

specifies the search space:

```
:~ assigned(V0, V1).[1@1, 1, V0, V1]
:~ assigned(V0, V1).[1@2, 2, V0, V1]
:~ type(V0, V1, c2).[1@1, 3, V0, V1]
:~ type(V0, V1, c2).[1@2, 4, V0, V1]
:~ assigned(V0, V1),
   assigned(V0, V2).[1@1, 5, V0, V1, V2]
:~ assigned(V0, V1),
   assigned(V0, V2).[1@2, 6, V0, V1, V2]
:~ assigned(V0, V1),
   assigned(V2, V1).[1@1, 7, V0, V1, V2]
:~ assigned(V0, V1),
   assigned(V2, V1).[1@2, 8, V0, V1, V2]
:~ assigned(V0, V1),
   assigned(V2, V3).[1@1, 9, V0, V1, V2, V3]
:~ assigned(V0, V1),
   assigned(V2, V3).[1@2, 10, V0, V1, V2, V3]
:~ assigned(V0, V1),
   type(V0, V1, c2).[1@1, 11, V0, V1]
:~ assigned(V0, V1),
   type(V0, V1, c2).[1@2, 12, V0, V1]
:~ assigned(V0, V1),
   type(V0, V2, c2).[1@1, 13, V0, V1, V2]
:~ assigned(V0, V1),
   type(V0, V2, c2).[1@2, 14, V0, V1, V2]
:~ assigned(V0, V1),
   type(V2, V1, c2).[1@1, 15, V0, V1, V2]
:~ assigned(V0, V1),
   type(V2, V1, c2).[1@2, 16, V0, V1, V2]
:~ assigned(V0, V1),
   type(V2, V3, c2).[1@1, 17, V0, V1, V2, V3]
:~ assigned(V0, V1),
   type(V2, V3, c2).[1@2, 18, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   assigned(V0, V3).[1@1, 19, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   assigned(V0, V3).[1@2, 20, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   assigned(V3, V1).[1@1, 21, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   assigned(V3, V1).[1@2, 22, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   assigned(V3, V2).[1@1, 23, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   assigned(V3, V2).[1@2, 24, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   assigned(V2, V3).[1@1, 25, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   assigned(V2, V3).[1@2, 26, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   assigned(V3, V1).[1@1, 27, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   assigned(V3, V1).[1@2, 28, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V0, V1, c2).[1@1, 29, V0, V1, V2]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V0, V1, c2).[1@2, 30, V0, V1, V2]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V0, V2, c2).[1@1, 31, V0, V1, V2]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V0, V2, c2).[1@2, 32, V0, V1, V2]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V0, V3, c2).[1@1, 33, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V0, V3, c2).[1@2, 34, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V3, V1, c2).[1@1, 35, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V3, V1, c2).[1@2, 36, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V3, V2, c2).[1@1, 37, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V0, V2),
   type(V3, V2, c2).[1@2, 38, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V0, V1, c2).[1@1, 39, V0, V1, V2]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V0, V1, c2).[1@2, 40, V0, V1, V2]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V0, V3, c2).[1@1, 41, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V0, V3, c2).[1@2, 42, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V2, V1, c2).[1@1, 43, V0, V1, V2]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V2, V1, c2).[1@2, 44, V0, V1, V2]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V2, V3, c2).[1@1, 45, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V2, V3, c2).[1@2, 46, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V3, V1, c2).[1@1, 47, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   type(V3, V1, c2).[1@2, 48, V0, V1, V2, V3]
```

```
:~ assigned(V0, V1), assigned(V2, V3),
   type(V0, V1, c2).[1@1, 49, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V3),
   type(V0, V1, c2).[1@2, 50, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V3),
   type(V0, V3, c2).[1@1, 51, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V3),
   type(V0, V3, c2).[1@2, 52, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V3),
   type(V2, V1, c2).[1@1, 53, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V3),
   type(V2, V1, c2).[1@2, 54, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V3),
   type(V2, V3, c2).[1@1, 55, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V3),
   type(V2, V3, c2).[1@2, 56, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V1),
   V0 != V2.[1@1, 57, V0, V1, V2]
:~ assigned(V0, V1), assigned(V2, V1),
   V0 != V2.[1@2, 58, V0, V1, V2]
:~ assigned(V0, V1), assigned(V2, V3),
   V0 != V2.[1@1, 59, V0, V1, V2, V3]
:~ assigned(V0, V1), assigned(V2, V3),
   V0 != V2.[1@2, 60, V0, V1, V2, V3]
:~ assigned(V0, V1), type(V2, V1, c2),
   V0 != V2.[1@1, 61, V0, V1, V2]
:~ assigned(V0, V1), type(V2, V1, c2),
   V0 != V2.[1@2, 62, V0, V1, V2]
:~ assigned(V0, V1), type(V2, V3, c2),
   V0 != V2.[1@1, 63, V0, V1, V2, V3]
:~ assigned(V0, V1), type(V2, V3, c2),
   V0 != V2.[1@2, 64, V0, V1, V2, V3]
```

3.2 Example Tasks

In this section, we give example learning tasks involving ordering examples encoded in ILASP. The first shows how ILASP can be used to learn soft constraints in a simple scheduling problem and is motivated by the running example in the paper [9]. Further examples may follow in later editions of this manual.

3.2.1 Scheduling

The following scheduling example task is motivated by the running example in the paper [9]. It shows how we can learn preferences using ILASP2 in the setting of interview scheduling.

Example 3.4. Consider the following learning task describing an interview scheduling problem. There are 9 slots to be assigned (3 each on 3 days) and each slot is for a candidate on one of two courses ($c1$ or $c2$).

```
slot(mon,1..3).     % Three slots each on Monday, Tuesday and Wednesday.
slot(tue,1..3).
slot(wed,1..3).



type(mon,1,c2).     % The following interview slots for c1 and c2 are to be assigned:
type(mon,2,c1).     %
type(mon,3,c1).     %            ---------------------
type(tue,1,c1).     %           |   | Mon | Tue | Wed |
type(tue,2,c1).     %            =====================
type(tue,3,c2).     %           | 1 | c2  | c1  | c1  |
type(wed,1,c1).     %           |---+-----+-----+-----|
type(wed,2,c2).     %           | 2 | c1  | c1  | c2  |
type(wed,3,c1).     %           |---+-----+-----+-----|
                    %           | 3 | c1  | c2  | c1  |
                    %            ---------------------


0 { assigned(X, Y) } 1 :- slot(X, Y).


#modeo(assigned(var(day), var(time)), (positive)).
#modeo(1, type(var(day), var(time), const(course)), (positive)).
#modeo(1, var(day) != var(day)).
```

```
#constant(course, c2).
#weight(1).


% For each example, we show the corresponding partial timetable.  "Yes" means
% that the person was definitely assigned the slot; "No" means that the person
% definitely was not assigned the slot; "?" meas that whether this slot was
% assigned to this person is unknown.

#pos(pos1, {          %          ---------------------
  assigned(mon,1)     %         |   | Mon | Tue | Wed |
}, { }).              %          =====================
                      %         | 1 | Yes |  ?  |  ?  |
                      %         |---+-----+-----+-----|
                      %         | 2 |  ?  |  ?  |  ?  |
                      %         |---+-----+-----+-----|
                      %         | 3 |  ?  |  ?  |  ?  |
                      %          ---------------------

#pos(pos2, {          %          ---------------------
  assigned(mon,3),    %         |   | Mon | Tue | Wed |
  assigned(wed,1),    %          =====================
  assigned(wed,3)     %         | 1 | No  |  ?  | Yes |
}, {                  %         |---+-----+-----+-----|
  assigned(mon,1),    %         | 2 |  ?  |  ?  | No  |
  assigned(tue,3),    %         |---+-----+-----+-----|
  assigned(wed,2)     %         | 3 | Yes | No  | Yes |
}).                   %          ---------------------

#pos(pos3, {          %          ---------------------
  assigned(mon,1),    %         |   | Mon | Tue | Wed |
  assigned(wed,3)     %          =====================
}, {}).               %         | 1 | Yes |  ?  |  ?  |
                      %         |---+-----+-----+-----|
                      %         | 2 |  ?  |  ?  |  ?  |
                      %         |---+-----+-----+-----|
                      %         | 3 |  ?  |  ?  | Yes |
                      %          ---------------------


#pos(pos4, {          %          ---------------------
  assigned(mon,1),    %         |   | Mon | Tue | Wed |
  assigned(mon,2),    %          =====================
  assigned(mon,3)     %         | 1 | Yes | Yes | Yes |
}, {                  %         |---+-----+-----+-----|
  assigned(tue,1),    %         | 2 | No  | No  | No  |
  assigned(tue,2),    %         |---+-----+-----+-----|
  assigned(tue,3),    %         | 3 | No  | No  | No  |
  assigned(wed,1),    %          ---------------------
  assigned(wed,2),
  assigned(wed,3)
}).

#pos(pos5, {          %          ---------------------
  assigned(mon,2),    %         |   | Mon | Tue | Wed |
  assigned(mon,3)     %          =====================
},{                   %         | 1 | No  | Yes | Yes |
  assigned(mon,1),    %         |---+-----+-----+-----|
  assigned(tue,1),    %         | 2 | No  | No  | No  |
  assigned(tue,2),    %         |---+-----+-----+-----|
  assigned(tue,3),    %         | 3 | No  | No  | No  |
  assigned(wed,1),    %          ---------------------
```

```
  assigned(wed,2),
  assigned(wed,3)
}).

#pos(pos6, {          %           --------------------
  assigned(mon,2),    %          |   | Mon | Tue | Wed |
  assigned(tue,1)     %           ====================
},{                   %          | 1 | No  | Yes | No  |
  assigned(mon,1),    %          |---+-----+-----+-----|
  assigned(mon,3),    %          | 2 | Yes | No  | No  |
  assigned(tue,2),    %          |---+-----+-----+-----|
  assigned(tue,3),    %          | 3 | No  | No  | No  |
  assigned(wed,1),    %           --------------------
  assigned(wed,2),
  assigned(wed,3)
}).

#cautious_ordering(pos4, pos3).
#cautious_ordering(pos2, pos1).
#cautious_ordering(pos2, pos3).
#brave_ordering(pos5, pos6).
#maxv(4).
#maxp(2).
```

As the positive examples are already covered by the background knowledge (and there are no negative examples), there is no need to change the answer sets of this program. For this reason, we only need to learn weak constraints which cover the brave and cautious orderings. When run with ILASP, this task produces the output:

```
:~ assigned(V0, V1), type(V0, V1, c2).[1@2, 1, V0, V1]
:~ assigned(V0, V1), assigned(V2, V3), V0 != V2.[1@1, 2, V0, V1, V2, V3]

Pre-processing  : 0.01s
Solve time      : 0.801s
Total           : 0.811s
```

The first, higher priority weak constraint says that this person would like to avoid having interview slots for the course c2. The second, lower priority weak constraint says that the person would like to avoid pairs of slots which occur on different days.

4 Specifying Learning Tasks with Noise in ILASP

Most real data has some element of noise in it. The tasks specified so far assume noise free data and, as such, require that all examples should be covered by any inductive solution.

In fact, ILASP does have a mechanism for dealing with noisy data. Any example (including ordering examples) can be given a weight w (a positive integer) to indicate that this example might be *noisy*. The goal of ILASP without noise is to search for an hypothesis H which covers all the examples such that $|H|$ is minimal. Any example which is not given a weight is considered *noise-free* and, as such, *must* be covered.

If we let W be the sum of the weights of examples which might be noisy and is not covered, then the goal of ILASP is now to find an hypothesis which covers all the noise-free examples and minimises $|H| + W$.

These weights are specified in ILASP by giving `example_name@weight` in place of the usual example name. If an example has been treated as noisy by the inductive solution which ILASP has found then it will report: `% Warning example: 'example_name' treated as noise.`

4.1 Example: Noisy Sudoku

Recall the sudoku task from section 2.3.1. If we now assign weights to the examples then the hypothesis might change as it might be more costly to cover an example than to pay the corresponding penalty. Note that the original hypothesis had length 26 with each of the constraints having length 3 and the choice rule having length 17.

Example 4.1. If we set the weights as:

```
#pos(a@10, {
  value((1, 1), 1),
  value((1, 2), 2),
  value((1, 3), 3),
  value((1, 4), 4),
  value((2, 3), 2)
}, {
  value((1, 1), 2),
  value((1, 1), 3),
  value((1, 1), 4)
}).

#neg(b@10, { value((1, 1), 1), value((1, 3), 1) },{}).
#neg(c@10, { value((1, 1), 1), value((3, 1), 1) },{}).
#neg(d@10, { value((1, 1), 1), value((2, 2), 1) },{}).
```

then ILASP returns:

```
% Warning: example 'a' treated as noise.

Pre-processing  : 0.032s
Solve time      : 0.135s
Total           : 0.167s
```

This means that the optimal solution is to return the empty hypothesis. This covers the three negative examples and so the only penalty paid is 10 for the single positive example.

Example 4.2. If we set the weights as:

```
#pos(a@100, {
  value((1, 1), 1),
  value((1, 2), 2),
```

```
  value((1, 3), 3),
  value((1, 4), 4),
  value((2, 3), 2)
}, {
  value((1, 1), 2),
  value((1, 1), 3),
  value((1, 1), 4)
}).

#neg(b@10, { value((1, 1), 1), value((1, 3), 1) },{}).
#neg(c@10, { value((1, 1), 1), value((3, 1), 1) },{}).
#neg(d@10, { value((1, 1), 1), value((2, 2), 1) },{}).
```

then ILASP returns:

```
:- value(V0,V1), value(V2,V1), same_col(V0, V2).
:- value(V0,V1), value(V2,V1), same_block(V2, V0).
:- value(V0,V1), value(V2,V1), same_row(V0, V2).
1 {value(V0,4); value(V0,3); value(V0,2); value(V0,1) } 1 :- same_row(V0, V1).

Pre-processing  : 0.027s
Solve time      : 1.857s
Total           : 1.884s
```

This means that the optimal solution is to return the optimal hypothesis from the original task (without noise).

Example 4.3. If we set the weights as:

```
#pos(a@100, {
  value((1, 1), 1),
  value((1, 2), 2),
  value((1, 3), 3),
  value((1, 4), 4),
  value((2, 3), 2)
}, {
  value((1, 1), 2),
  value((1, 1), 3),
  value((1, 1), 4)
}).

#neg(b@1, { value((1, 1), 1), value((1, 3), 1) },{}).
#neg(c@4, { value((1, 1), 1), value((3, 1), 1) },{}).
#neg(d@4, { value((1, 1), 1), value((2, 2), 1) },{}).
```

then ILASP returns:

```
% Warning: example ‘b’ treated as noise.
:- value(V0,V1), value(V2,V1), same_block(V0, V2).
:- value(V0,V1), value(V2,V1), same_row(V2, V0).
1 {value(V0,4); value(V0,3); value(V0,2); value(V0,1) } 1 :- same_row(V0, V1).

Pre-processing  : 0.032s
Solve time      : 3.191s
Total           : 3.223s
```

Note that as examples `c` and `d` have weight 4 it is less costly to learn their corresponding constraints (of length 3); whereas, because example `b` only has weight 1, it is less costly to treat this example as noise and pay the penalty than to learn an extra constraint of length 3.

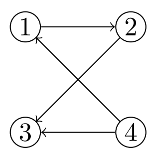
ILASP3, which is targeted at running noisy tasks with large numbers of examples, and may be much faster than ILASP2 or ILASP2i on these types of task.

5 Context Dependent Examples

Until now, we have focused on finding a hypothesis H such that H explained the examples using one fixed background knowledge B. In some settings, however, it is useful to be able to express that in one context, $B \cup H$ should explain some examples, and in another context, it should explain other examples. Recently, we introduced the notion of a *context dependent* partial interpretations [11]. The context of an example is an extra bit of background knowledge, which only applys to a specific example. Context dependent examples are implemented in version 2.6 of ILASP.

5.1 Hamiltonian Graphs Example

One learning task which can make use of these context dependent examples is learning the definition of whether or not a graph contains a Hamilton cycle (i.e. whether it is Hamiltonian). The four node Hamiltonian graph G in figure 1 can be represented by the set of facts F_G. We first show how this can be represented without context, in ILP_{LOAS}; and then show a much simpler representation using context dependent examples. Interestingly, context dependent examples do more than simplify the representation: they provide useful information to ILASP when solving. ILASP2i is designed to take advantage of this information, and solves the context dependent task much faster than the non context dependent one.



$$F_G = \begin{cases} \texttt{node(1..4).} \\ \texttt{edge(1,2).} \\ \texttt{edge(2,3).} \\ \texttt{edge(3,4).} \\ \texttt{edge(4,1).} \end{cases}$$

Figure 1: An example of a Hamiltonian Graph G and its corresponding representation in ASP, F_G.

To decide whether a graph is Hamiltonian or not, we can use the program H below:

```
reach(V0) :- in(1,V0).
reach(V1) :- in(V0,V1), reach(V0).
0 {in(V0,V1) } 1 :- edge(V0, V1).
:- node(V0), not reach(V0).
:- in(V0,V1), in(V0,V2), V1 != V2.
```

If for a graph G and its corresponding set of facts F_G, G is Hamiltonian if and only if $F_G \cup H$ is satisfiable. In Hamilton A and Hamilton B, we give two alternative encodings of a task which aims to learn H: the first in ILP_{LOAS} and the second in $ILP^{context}_{LOAS}$. Both learning tasks have the same hypothesis space, consisting of 104 normal rules, choice rules and hard constraints.

5.1.1 Hamiltonian graphs in ILP_{LOAS} : non-context dependent examples

In an ILP_{LOAS} representation of the learning task, the background B knowledge needs to generate the possible graphs of size $1, \ldots, 4$. B is as follows.

```
1 { size(1); size(2); size(3); size(4) } 1.
0 { edge(N1, N2) } 1 :- node(N1), node(N2).

node(1).
```

```
node(2) :- size(S), S > 1.
node(3) :- size(3).
node(3) :- size(4).
node(4) :- size(4).
```

Our examples then contain details of which edges are in (and not in) each graph. For example, one positive example is:

```
#pos({
  size(3),
  edge(1,3),
  edge(2,1),
  edge(2,3),
  edge(3,2)
}, {
  edge(1,2),
  edge(3,1)
}).
```

An ILP_{LOAS} task for this learning task can be downloaded from `https://www.doc.ic.ac.uk/~ml1909/ILASP/benchmarks/hamiltonA.las`

5.1.2 Hamilton B: context dependent examples

We now demonstrate how to make use of the context dependent examples in $ILP_{LOAS}^{context}$. We don't need any background knowledge (it is empty), and instead encode the graphs in the contexts of examples such as in the positive example:

```
#pos({}, {}, {
  node(1..3).
  edge(1,2).
  edge(2,1).
  edge(2,3).
  edge(3,1).
}).
```

An $ILP_{LOAS}^{context}$ task for this learning task can be downloaded from `https://www.doc.ic.ac.uk/~ml1909/ILASP/benchmarks/hamiltonB.las`

5.2 Context Dependent Ordering Examples

We also support using context dependent partial interpretations in ordering examples. The ordering examples refer to the ID's of the context dependent partial interpretations in the same way as with the non context dependent partial interpretations.

5.2.1 Note on weak constraints in contexts

We do not support using weak constraints in contexts. This is because we do not see a use case for an ordering example $\langle e_1, e_2 \rangle$ such that e_1 and e_2 are compared on different weak constraints.

6 Hypothesis length

When specifying the search space in full you must also specify the length of each rule. This length is used to determine the length of each hypothesis. The goal of ILASP is to search for an inductive solution which minimises this length.

If you use mode declarations to specify your search space then ILASP will assign a length to each rule. Usually, the length of a rule R (written $|R|$) is just the number of literals in the rule. The only exceptions to this are choice rules. The aggregate in the head of a choice rule is not counted simply as the number of atoms it contains, as this tends to (in our opinion unreasonably) favour learning hypotheses containing choice rules.

A counting aggregate $\mathtt{l\{h_1, \ldots, h_n\}u}$ has length

$$n \times \sum_{i=l}^{u} {}^{n}C_i$$

The inuition behind this is that this is the length of the counting aggregate once it has been converted to disjunctive normal form.

Example 6.1. The counting aggregate $\mathtt{1\{p, q\}2}$ represented in disjunctive normal form would be $(\mathtt{p} \wedge \mathtt{not\ q}) \vee (\mathtt{not\ p} \wedge \mathtt{q}) \vee (\mathtt{p} \wedge \mathtt{q})$; so, this is counted as length 6 rather than just length 2.

In our opinion, and for the examples we have tried, this weighting of choice rules better reflects the added complexity they bring to an hypothesis.

If you wish to use a our search space, but with a different measure of hypothesis length then our suggested solution is to call ILASP with the option `-s` which will produce our search space with our weights. You can then edit the weights and add the rules to the search space manually (omitting the original mode bias).

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