A Proof from PROTEIN on msc006$_1$.*complete*[*]

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July 18, 1997

Axiom 0.1 (*not_total*(p)) $\neg\ p(a,b)$.

Axiom 0.2 (*all_related*(p,q)) $p(A,B) \vee q(A,B)$.

Axiom 0.3 (*symmetric*(q)) $q(A,B) \rightarrow q(B,A)$.

Axiom 0.4 (*transitive*(q)) $q(A,B) \wedge q(B,C) \rightarrow q(A,C)$.

Axiom 0.5 (*transitive*(p)) $p(A,B) \wedge p(B,C) \rightarrow p(A,C)$.

Theorem 0.1 $q(c,d)$.

Proof[1]. We show directly that

$$q(c,d). \tag{1}$$

We show indirectly that

$$q(c,b). \tag{2}$$

Let us assume that

$$\neg\ q(c,b). \tag{3}$$

Because of *all_related*(p,q) and by *not_total*(p) $q(a,b)$. Hence by *symmetric*(q)

$$q(b,a). \tag{4}$$

By (3) and by *all_related*(p,q) $p(c,b)$. Hence by *transitive*(p) and by *not_total*(p) $\neg\ p(a,c)$. Hence by *all_related*(p,q) $q(a,c)$. Hence by *transitive*(q) and by (4) $q(b,c)$. Hence by *symmetric*(q) $q(c,b)$. This contradicts (3). Thus we have completed the proof of (2).

We show indirectly that

$$q(b,d). \tag{5}$$

[*]This manuscript was generated by ILF. The development of ILF was supported by the Deutsche Forschungsgemeinschaft. For information on ILF contact gehne@mathematik.hu-berlin.de.

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Let us assume that

$$\neg\ q(b,d). \tag{6}$$

Because of $all_related(p,q)$ and by $not_total(p)$ $q(a,b)$. Hence by $symmetric(q)$

$$q(b,a). \tag{7}$$

By (6) and by $symmetric(q)$ $\neg\ q(d,b)$. Hence by $all_related(p,q)$ $p(d,b)$. Hence by $transitive(p)$ and by $not_total(p)$ $\neg\ p(a,d)$. Hence by $all_related(p,q)$ $q(a,d)$. Hence by $transitive(q)$ and by (7) $q(b,d)$. This contradicts (6). Thus we have completed the proof of (5).

Hence by $transitive(q)$ and since $q(c,b)$ $q(c,d)$. This completes the proof of (1).

q.e.d.