A Proof from PROTEIN*

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Abstract

This is just an experiment.

We assume that the reader is acquainted with the elementary theory described in the appendix.

Axiom 0.1 (The size of the successor) $A < A^+$.

Axiom 0.2 (The fact that each number is even or odd) *A is even* $\vee$ *A is odd.*

Axiom 0.3 (The fact that the successor of an odd number is even) *A is odd* $\rightarrow$ A^+ *is even.*

Theorem 0.1 $\exists\ x_1\ (x_1$ *is even* $\wedge\ a < x_1)$.

Proof[1]. We show indirectly that

$$\exists\ x_1\ (x_1 \text{ is even} \wedge a < x_1). \tag{1}$$

Let us assume that

$$\forall\ x_1\ (x_1 \text{ is not even} \vee a \not< x_1). \tag{2}$$

Hence by the size of the successor a^+ is not even. Hence by the fact that each number is even or odd a^+ is odd. Hence by the fact that the successor of an odd number is even $a^{+\,+}$ is even. Because of the size of the successor $a < a^{+\,+}$. Therefore by (2) we have a contradiction.

This completes the proof of (1).

q.e.d.

Appendix

- $A < B \rightarrow B \not< A$
- $A < A \rightarrow false$
- $A < B \wedge B < C \rightarrow A < C$
- $A \not< B \wedge C < B \rightarrow A \not< C$

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